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ABSTRACT

The relationships of intelligence, achievement, age, sex, heredity, and psychosocial factors to mathematical creativity are carefully reviewed and discussed in this document. A bibliography listing 125 references is included. (Author/DT)

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MATHEMATICS EDUCATION REPORTS

ABILITY AND CREATIVITY IN MATHEMATICS

by

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Preface

Although mathematics educators are often most concerned with developing problem-solving and creative behaviors in mathematics students, many studies of mathematical abilities are limited to lower levels of the cognitive domain such as recall or computation.

This research review represents an attempt to ferret out and focus upon abilities that interact with creativity in mathematics. The relationships of intelligence, achievement, age, sex, heredity, and psychosocial factors to mathematical creativity are carefully reviewed and discussed. The result should not only clarify the present status of research knowledge in this area, but should also suggest implications for new research studies.

Jon L. Higgins
Editor

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ABILITY AND CREATIVITY IN MATHEMATICS

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An earlier paper by the writer dealt briefly with intellective factors in learning mathematics, or, more precisely, with the structure and correlates of mathematical ability (Aiken, 1971). As a follow-up to that paper, an extensive review of the literature on language factors and mathematics learning was conducted, concluding, like the first paper, with specific recommendations for further research (Aiken, 1972). The present paper is the third in a series of critical reviews of research in the cognitive domain of variables that influence mathematical ability. Although much of the emphasis in the discussions of ability and creativity presented here is on cognitive variables, more attention has been given elsewhere to research in the affective domain (Aiken, 1970). In any event, affective and cognitive variables interact in a complex way to influence performance in mathematics. Therefore, it is important to consider the effects of attitudes, temperament, and other personality variables, as well as the sociocultural context, in discussing mathematical ability and creativity.

Types and Factors of Mathematical Ability[1]

The concept of mathematical "types" or dispositions has been promoted by laymen and professionals alike. For example, it is not uncommon to

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[1]Werdelin (1958) defines mathematical ability as "the ability to understand the nature of mathematical (and similar) problems, symbols, methods and proofs; to learn them, to retain them in the memory, and to reproduce them; to combine them with other problems, symbols, methods and proofs, and to use them when solving mathematical (and similar) tasks."

hear a student explain his poor performance in mathematics as being due to the lack of a "mathematical mind." Similarly, the Russian educational psychologist Krutetskii (1969c) refers to children who are gifted in this subject as having a "mathematical frame of mind." According to Krutetskii, such children find mathematical meanings in many aspects of reality, and tend to categorize things in terms of mathematics and logic.

Logical and Intuitive Minds.

In contrast, or perhaps supplementary, to Krutetskii's notion of a "mathematical frame of mind" is the idea that there are several different types of mathematical minds. In a survey of the educational philosophies of fourteen eminent mathematicians, Carlton (1959) found that the writings of many of these famous men refer to more than one kind of mathematical mind. One type of mind is said to be logical and formal, whereas another is more intuitive; one type is fast, and another is slow. The distinction between these types of mathematical mind was dealt with most extensively by Poincaré (1952), who maintained that geometers are more intuitive and analysts more logical in their thinking. Recent writings on mathematical ability have also contrasted the logical ("reflective") and the intuitive modes of intellectual ability.

Factor Analyses of Mathematical Abilities

The concept of mathematical types, although appealing in many respects, has been criticized by both mathematicians and psychologists as an oversimplification. British and American psychologists in particular have applied the methods of factor analysis to the study of mathematical ability.

Noteworthy, and apparently uncontested, is the finding that children who excel in mathematics tend to score high on tests of general intelligence (Duncan, 1961; Kennedy and Walsh, 1965; Geng and Mehl, 1969). On the other hand, the existence of a broad group factor of mathematical ability has been a subject of controversy. British factor analysts (Blackwell, 1940; Barakat, 1951; Wrigley, 1958) have argued that there is a group factor of mathematical ability, defined as quantitative reasoning or problem-solving ability. Such a factor is said to have substantial loadings on arithmetic and mathematics tests of various kinds. However, American psychologists (Guilford, Hoepfner, and Petersen, 1965; Hills, 1957) have questioned the notion of a general mathematical ability factor and have tended to view mathematical achievement as influenced by a composite of specific abilities. Although factor-analytic research has revealed a number of mathematical ability factors, these findings are not necessarily irreconcilable with the notion of a general mathematical group factor. In the list presented in Table 1 of the most salient mathematical factors that have been extracted in various investigations, the "Deductive (general) reasoning" factor is actually very close to the mathematical group factor espoused by British psychologists.[2]

Table 1

Representative Mathematical Ability Factors Obtained in Various Factor-Analytic Investigations

Deductive (general) reasoning - (Blackwell, 1940; Kline, 1960; Very, 1967; Werdelin, 1958, 1966)
Inductive reasoning - (Werdelin, 1958)
Numerical ability - (Kline, 1960; Very, 1967; Werdelin, 1958, 1966; Wooldridge, 1964)
Spatial-perceptual ability - (Blackwell, 1940; Very, 1967; Werdelin, 1966)
Verbal comprehension - (Blackwell, 1940; Kline, 1960; Very, 1967; Werdelin, 1958, 1966; Wooldridge, 1964)

[2]Coleman (1956) also noted that mathematical ability is closely related to deductive reasoning ability.

Since the samples employed in various factor-analytic investigations vary in age, sex, sociocultural variables, and in other ways, the particular factors extracted and their loadings on different mathematics tests may be inconsistent from study to study. Therefore, the findings of different investigations are not always comparable. For example, the factors listed in Table 1 were not the only ones obtained in these investigations. Among the other factors reported were algebraic manipulation skill, adaptability to a new task (Kline, 1960), and algebra achievement (Wooldridge, 1964).

Krutetskii's Components of Mathematical Ability

Also of interest to research concerning the analysis of mathematical ability are the "components of mathematical ability" which Krutetskii (1966) has isolated from his observations and impressions of the responses of Russian schoolchildren to mathematics problems. Based on logical analysis rather than factor analysis, these basic components are: (a) formalized perception of mathematical material, (b) generalization of mathematical material, (c) "curtailment" of thought, (d) flexibility of thought, (e) striving for economy of mental forces, (f) mathematical memory, and (g) spatial concepts.[3]

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[3]It is interesting that not all of the abilities isolated by either factor analysis or logical analysis as important to performance in mathematics were possessed to a high degree by certain famous mathematicians. Thus, many mathematicians have not been noted for their verbal fluency, good memory, or computational skill (see Krutetskii, 1969c and Smith, 1964). Also, from a study of college students enrolled in a two-year sequence in undergraduate mathematics, Coleman (1956) concluded that a good memory is not essential to mathematical ability, the ability to see spatial relationships is helpful in only a limited area, and computational ability is only remotely related to ability in the subject.

Relationships of Mathematical Factors to Achievement

A logical sequel to the isolation of several mathematical factors is the construction of tests to measure each of the factors. Scores on these tests might then be used to predict performance in specific mathematics courses and assist in the planning of instructional treatments suitable for specific patterns of abilities. Although it is reasonable to expect that scores on tests of specific mathematical ability factors would surpass general intelligence test scores as predictors of success in mathematics, the actual findings have frequently been otherwise. Factored tests, such as those in Guilford's structure-of-intellect battery, have occasionally demonstrated superior ability in predicting achievement in general mathematics, algebra, and calculus (Guilford, Hoepfner, & Petersen, 1965; Hills, 1957). But the results of many other investigations show that previous mathematics grades are the best predictor of later mathematics achievement and that the correlations between the latter variable and scores on factored tests of mathematical ability are usually rather low (Wick, 1965).

Aptitude-Treatment Interactions.

A number of attempts have been made to design instructional procedures that are better suited to learners having specific patterns of mathematical factors or abilities. The majority of these investigations have not been very successful (Becker, 1967; Carry, 1968; also see Aiken, 1971), but partial success has been obtained in selected instances (Behr, 1967; King, Roberts, & Kropp, 1969). For example, Behr (1967) employed two treatment programs to teach modulus seven arithmetic to college

students. One treatment consisted of presenting the lessons in a verbal mode, whereas the other treatment employed figures and diagrams. Several instances of interaction between aptitude factors (semantic vs. figural) and instructional method (verbal vs. figural) were observed.

Obviously, no clear picture of the structure of mathematical abilities has emerged from studies employing either logical analysis or factor analysis. Not surprisingly, Kose (1967) concluded that the contributions of factor analysis to an understanding of mathematical abilities have been relatively small compared to the great amount of work in this area. In any case, the expectation that factor analysis would reveal a set of fixed, genetically-determined mental entities has not been confirmed. One reason for the inconclusiveness of the findings employing factor-analytic techniques is that a multitude of methodological and organismic variables can affect the results of a factor analysis. Among the organismic variables, those of age and sex have received the greatest attention, a subject to which we shall return later. For the moment, however, let us shift the focus of this paper to a consideration of a topic which has been the subject of extensive theoretical and practical attention during the past decade or so. Increased interest in this topic — that of creativity — is reflective of the growing disenchantment with measures of general intelligence and the educational and social consequences of applying these measures.

The Meaning and Measurement of Creativity

Highly gifted or creative people, such as Archimedes, Newton, and Gauss, have always been the objects of wonder. Some of the questions concerning creativity, and especially creativity in mathematics, that have been asked by researchers during the past few years are: What is creativity? How can

creativity be measured? Is creativity in mathematics distinct from creativity in other areas, or is there a general trait of creativity that overlaps many fields? Is creative ability inborn, or, if it can be developed, what is the best way to go about it? These questions are not unique to the topic of creativity, and, as with other psychological variables, final answers are not easily obtained. The remainder of this paper will focus on research designed to provide at least partial answers to these questions and others concerned with creativity in mathematics.

Defining Creativity and Creative Thinking

Getzels (1969) classifies definitions of creativity into three categories: those that refer to (a) a manifest product which is novel and useful (MacKinnon, 1962); (2) an underlying process which is divergent but fruitful (Ghiselin, 1952); (3) a subjective experience which is inspired and immanent (Maslow, 1963). With respect to mathematical creativity in particular, the following definitions have been offered:

1. Creative thinking is thinking which results in additions to knowledge (Carlton, 1959).
2. Creativity is the ability to produce original or unusual, applicable methods of solution for problems in mathematics (Spraker, 1960).
3. Creativity is the ability to combine ideas, things, techniques, or approaches in a new way (Romey, 1970).
4. Creative mathematics is the ability to analyze a given problem in many ways, observe patterns, see likenesses and differences, and on the basis of what has worked in similar situations decide on a method of attack in an unfamiliar situation (Laycock, 1970).

The first two definitions are concerned with the creative product, and the last two with the creative process. Not encompassed by these four definitions is the subjective experience ("flash of insight") aspect of creativity, which has been vividly described by Poincaré (1952), Hadamard (1954), and others.

Stages in Creative Thinking. Both Poincaré and Hadamard maintained that much of the work that goes on in creative mathematical thinking is unconscious or intuitive, rather than strictly logical, formal, and conscious. But these two mathematicians and later investigators recognized that there are actually several stages in creative thinking. The first stage is a period of preparation, concentration, and deep involvement in a problem. This is followed by a second stage — incubation, during which time the problem is set aside for a while but work on a solution continues at the unconscious level. If the prolonged unconscious work of the second stage is successful, a third stage occurs — illumination or insight into a solution. During the fourth and final stage, verification, elaboration, and refinement of the insightful solution takes place (see Taylor, 1963 and Skemp, 1971).

Convergent and Divergent Thinking. Certain definitions of creative thinking emphasize its divergent, or open-ended, nature. In contrast to convergent thinking, the goal of which is to find the one absolutely correct solution to a problem, problems requiring divergent thinking have many possible solutions. On measures of convergent thinking ability, such as conventional intelligence tests, examinees are scored according to the correctness of their answers. But on tests of divergent thinking, a person is scored on the originality of unusualness of his solutions to problems. Guilford (1964) maintains that the ability to think divergently and to transfer information are essential to creativity. He states that the creative

individual must be fluent, flexible, and capable of elaborating and redefining problems. According to Lovell (1968), however, divergent thinking ability is not a unitary factor. A person may be creative or capable of divergent thinking to a greater degree in one situation or with one type of task than another. For this reason, Lovell questions the existence of a general trait of creativity (also see Thorndike, 1963).

Measures of General Creativity

Various procedures have been applied to the measurement of creativity in general, including ratings by self and others; scales or tests of personality, motives, originality, and creativity; interest inventories; and tests of achievement, intelligence, and special abilities. In addition to the more traditional ratings of originality obtained from responses to inkblots, pictures, and other projective techniques, the most frequently used commercially-available tests of creativity are the Remote Associates Test (by S. A. & M. T. Mednick; Houghton Mifflin), the AC Test of Creative Ability (Buhl, 1960), the University of Southern California Tests (J. P. Guilford et al.; Sheridan Psychological Services), and the Torrance Tests of Creative Thinking (E. P. Torrance; Personnel Press). The last two of these are actually batteries of tests. Among the tests on the Guilford battery are Alternate Uses, Consequences, Decorations, Making Objects, Match Problems, Possible Jobs, and Sketches, in addition to the four Christensen-Guilford Fluency Tests. Each of the four forms of the Torrance tests contains seven Verbal Tests ("Thinking Creatively with Words") and three Figural Tests ("Thinking Creatively with Pictures"). In addition to these and other commercially-available instruments, Davis (1971) lists 15 noncommercially-available tests and eight attitude surveys for studying creative behavior and talent. These tests consist of a

variety of tasks designed to assess the examinee's divergent thinking abilities by determining the richness and originality of his ideas concerning various figures, arrangements of things, outcomes of events, word or sentence constructions, stories, and the like.

Measures of Mathematical Creativity

Many investigations of creativity in mathematics have employed one or more of the instruments referred to above. Several instruments have been designed, however, for the specific purpose of assessing creativity in mathematics. Meyer's (1970) measure of mathematical creativity in first graders employs behavioral ratings on six criteria. Five of these criteria describe activities engaged in by a creative person attempting to solve a problem, and the sixth criterion describes the results of his actions. Foster (1970) constructed two tests (A and B) to measure the creative abilities in mathematics of primary school children (ages 9 to 11 years). On the first test the examinee selects from a deck six cards having some attribute in common, the score being the number of sets constructed in five minutes. The second test requires the examinee to find as many sums as possible, using the numbers 2, 3, and 6 and the four fundamental operations.

Evans's (1965) tests were designed to measure the abilities of pupils in late elementary and early junior high school to respond creatively in mathematical situations. Each of the 16 tests consists of a mathematical situation, to which the examinee is asked to respond in as many ways as possible. These responses are scored for fluency (number of responses), flexibility (number of different kinds of responses), and originality (degree of uncommonness of responses). Similarly, Spraker's (1960) test for seventh graders requires the examinee to solve each of 31 mathematics problems in as many different

ways as he can. The scores on this test consist of the sum of numerical weights (1 to 4) assigned to responses according to their infrequency, the most common responses being given the smallest weights. Also designed for seventh graders is Prouse's (1967) creativity test, composed of ten items based on characteristics attributed to mathematics students who were judged to possess creative potential in the subject. Seven of these items measure divergent thinking ability, and the other three measure convergent thinking ability.

At least two tests, Buckeye's (1970) Test of Creative Ability in Mathematics and Mainville's (1972) Mathematical Creativity Test, have been designed for use with college students. Buckeye's test, consisting of six divergent thinking items, is scored for fluency alone. On the other hand, each of the two forms (A and B) of Mainville's test, composed of five divergent thinking items, is scored for both fluency and originality.

Relationships of Creativity to Other Intellective Variables

Since one of the reasons given for the construction of measures of creativity is that such instruments tap different psychological dimensions than conventional tests of intelligence or achievement, empirical studies of the correlations among these variables have been plentiful. A wide range of correlations between creativity and IQ, varying with the particular instruments and the group tested, has been reported. Consistent with earlier findings, Lanier (1967) and McGannon (1972) found the correlations of certain commercially-available tests of general creativity (Remote Associates Test, Torrance Tests of Creative Thinking) with intelligence test scores to be low (.20 to .40). In contrast, Banghart and Spraker (1963), Prouse (1967), and Evans (1965) reported moderate correlations (.40 to .60) between intelli-

gence and their measures of mathematical creativity. Although Borgen (1971) found the creativity tests prepared by Guilford and his associates to be significantly related to arithmetic achievement, the correlations between scores on Torrance's tests and mathematics achievement are usually rather low (Cicirelli, 1965; Lanier, 1967). Finally, Evans (1965) and Banghart and Spraker (1963) also found significant correlations between their mathematical creativity measures and tests of achievement in mathematics.

It may be concluded from these diverse findings that, with the possible exception of Guilford's tests, the correlations of both general intelligence and mathematical creativity with mathematics achievement are higher than those between the last variable and general creativity. Furthermore, since achievement in mathematics is significantly related to scores on general intelligence tests, it is not particularly surprising that Guilford's tests, which are also positively correlated with general intelligence (Wallach & Kogan, 1965) are related to mathematics achievement.[4]

The low correlations of general creativity with intelligence and academic achievment were not unexpected. Thus, certain psychologists maintain that intelligence tests, especially those requiring speeded solutions, are loaded against more creative students (Klein & Kellner, 1967). Furthermore, Bentley (1966) states that tests of intelligence and achievement favor memory and cognitive abilities and consequently penalize highly creative students. Another possible cause of the low correlations between creativity and traditional tests of ability or achievement is the moderate

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[4]Scores on the creativity tests prepared by Guilford and his associates show increases with age and grade level similar to those obtained with general intelligence tests (See Borgen, 1971).

reliability of most creativity tests, typically in the .80's. Also, the correlations among different measures of creativity are themselves frequently low, a fact that helps explain conflicting results obtained in studies employing different instruments.

British psychologists concerned with the relationships between creativity, intelligence, and achievement have tended to interpret the correlations among tests of intelligence, mathematics achievement, and general creativity as indicative of an underlying factor of general ability (Burt, 1962; Richards & Bolton, 1971). Yamamoto (1965) also concluded that the evidence from factor analysis does not confirm the existence of a separate creativity factor. Thorndike (1963), however, found some evidence for a broad factor of creativity separate from general intelligence. In any event, the debate over the relationship of creativity to intelligence and achievement continues, and other explanations have been offered. Thus, Evans (1965) states that above-average intelligence is necessary but not sufficient for high performance on his tests of mathematical creativity. Related to Evans' conclusion is Anderson's (1960) ability gradient theory. Applied to the subject of mathematics, this theory holds that there is a threshold level of intelligence (IQ = 120?) beyond which creative ability is more important than I. Q. in its effect on performance in the subject.

Heredity and Development in Mathematical Abilities

Intelligence and achievement are not the only variables related to mathematical creativity. Numerous organismic, personality, and sociocultural factors have been studied as causal or contingency variables in investigations of mathematical ability. In the next two sections of this paper, some of the physiological and psychosocial factors that have been

associated with mathematical ability and creativity will be considered.

Early Development and Heredity

The precociousness of many famous mathematicians is well-known and a subject of wide interest and discussion. Illustrative of this early giftedness are Blaise Pascal, Karl Friedrich Gauss, Jakob and Johann Bernoulli, Norbert Wiener, Charles Fefferman, and many other deceased and living mathematicians. Observations and surveys have revealed that mathematically gifted children tend to develop at a very early age (3 to 5 years according to Krutetskii, 1969b). Also, mathematicians obtain their Ph.D.'s earlier than other scientists (Super & Bachrach, 1957), and creative mathematicians are usually well advanced in research by their third decade of life (Albert, 1957). Obviously, early interest and development is not unique to creativity in mathematics, but it appears to be especially characteristic of this field. To be sure, there are exceptions to the general rule that mathematical geniuses manifest their ability in this subject as young children, e. g., Isaac Newton, Giuseppi Peano, and mathematical giftedness is not always recognized in young people, e.g., Evariste Galois (See Prielipp, 1969).

The fact that signs of mathematical talent appeared quite early in the lives of many famous mathematicians and that some of these men pursued their mathematical studies and research in spite of parental opposition has led to speculation concerning the role of hereditary determinants in mathematical ability. Révész (1940), for example, maintained that there are hereditary aspects to mathematical talent which develop before the age of sixteen. More recently, Stafford (1972) has interpreted higher mean scores for males and different correlations between same-sex and opposite-sex parent and child on tests of quantitative ability as evidence for the transmission of these

abilities by sex-linked, recessive genes.

Although the influence of heredity on mathematical abilities is debatable, at least one psychologist has offered a tentative physiological explanation for individual differences in these abilities. Krutetskii (1969 a, b) suggested that mathematical stimuli and activities affect the capacity or strength of nervous processes in a different way than other kinds of stimuli or activities. In addition, the nervous systems of some people may be more sensitive to stimuli having mathematical characteristics (relations, symbols, numbers) than to other stimuli. In other words, their brains are more easily "oriented" toward relationships and symbols and respond optimally to these kinds of stimuli. Consequently, these people can form associations involving mathematics with less effort and retain them longer; they are also more resistant to fatigue that is specific to mathematical activity.

Age and Sex Differences in Mathematical Abilities

As was noted above, the existence of sex differences in mathematical abilities has led to considerable speculation concerning the relative roles of heredity and environment in determining these abilities. Research results show that, on the average, girls tend to score higher than boys on tests of verbal ability, arithmetical fundamentals, and rote memory, whereas boys are superior in spatial ability, arithmetic reasoning, and problem solving (see Aiken, 1971; Werdelin, 1961). But sex differences in abilities are less pronounced in the early grades, and there is a general differentiation of abilities with age and experience.

Age Differentiation Hypothesis. The occurrence of changes in mathematical abilities with age, in the direction of greater crystallization or specificity, is widely documented (Krutetskii, 1966; Very, 1967; Dye & Very, 1968; Geag & Mehl, 1969). This observed fractionation of mental abilities as the individual matures was labeled by Dye and Very (1968) as the "Age Differentiation Hypothesis," a hypothesis that has stimulated further research (e.g., Khan, 1970). Not only is the degree of age differentiation in mathematical abilities greater for boys than for girls (Dye & Very, 1968), but it varies with the particular social or cultural group studied. Anastasi (1970) has interpreted these findings in terms of Ferguson's (1956) hypothesis that specific mental abilities result from transfer of training among the different tasks required of people in different sociocultural contexts. For example, the relatively greater emphasis in Western culture on the acquisition of verbal skills by girls, as opposed to quantitative and spatial-perceptual skills, may help explain why Dye and Very (1968) obtained a well-differentiated verbal factor but less distinct quantitative and spatial factors with girls than with boys.

Piaget on Mathematical Development. The empirical research of Krutetskii (1969 a, b, c), Dye and Very (1968), and others has led to further investigation and theorizing, but a much greater impact on research in mathematical development has been exerted by Jean Piaget's theories of cognitive development. Piaget's conceptualization of intellectual development in terms of processes (assimilation and accommodation in the formation of mental schemas) and invariant stages or periods (period of sensorimotor operations, preoperational period, period of concrete operations, period of formal operations) has been applied to mathematical development by Piaget and others (Piaget, 1952; Piaget, Inhelder, & Szeminska, 1960; Lovell, 1968; Skemp, 1971). Piaget

maintains that the child passes through three stages in attaining the concept of number— nonconservation stage, transitional stage, and conservation stage. It is only during the third (conservation) stage, which usually begins at about the time he enters school, that the number of elements in a group does not vary with their arrangement (Piaget, 1952). Likewise, Piaget views the development of geometric conceptions as taking place in three stages: (1) attainment of topological concepts; (2) attainment of projective concepts; (3) attainment of Euclidean Concepts (Piaget, Inhelder & Szeminska, 1960).[5]

Research evidence obtained by other investigators does point to age trends in mathematical development, but because of the wide range of individual differences in rate of development Piaget's theories have not been fully substantiated (Dodwell, 1960; Sealey, 1960; Lovell, 1968), Consequently, redesigning mathematics curricula according to Piaget's developmental stages is undoubtedly premature and risky. Also, Lovell (1968) has found that gifted pupils (IQ = 140+) do not reach the periods of concrete operational or formal operational thought appreciably earlier than children of average intelligence. At both of these stages, however, the thought patterns (schema) of gifted children are much more flexible, being applied to a greater variety of situations than in the case of children of average ability.

Psychosocial Factors in Mathematical Ability and Creativity

Personality Correlates of Mathematics Achievement

As described in a recent paper by the writer (Aiken, 1970), numerous correlational studies concerned with the relationships of attitudes, anxiety,

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[5] Dienes (1963) identifies three substages or states within Piaget's stage of concrete operations: (1) a preliminary group state of random exploratory activity; (2) more structured state in which events are recognized as following certain events, leading to an insight concerning the whole operational pattern; and (3) an intense desire to use the new insight.

and other affective variables to achievement in mathematics have been conducted during the past 20 years or so. Since only about 50% of the score variance on measures of mathematical achievement can be accounted for by tests of general and special abilities, it seems reasonable to expect that scores on personality tests and inventories would improve the predictability of mathematics achievement. Unfortunately, due in no small degree to the limitations of psychological measuring instruments, affective measures have not usually contributed substantially to the predictive effeciency of cognitive variables. Nevertheless, it is recognized that personality traits do affect success in mathematics (e.g., Coleman, 1956).

The personality variables that have been found to have modest positive relationships to grades in mathematics courses and/or scores on mathematics achievement tests point to a greater sense of responsibility, more independence, a higher status, and generally effective psychosocial adjustment for high achievers in the subject (Aiken, 1963; Cleveland & Bosworth, 1967). Students who are more successful in mathematics are also less impulsive and more reflective than low achievers.

Professional Mathematicians. In a study of the personalities of professional mathematicians, compared with other scientists, Helson and Crutchfield (1970a) found lower scores for the former group on measures of "assertive self-assurance" but higher scores on "adaptive autonomy" and "humanitarian conscience." The mathematicians were characterized as high on order but low on the need for change; they were reserved, sensitive, conscientious, and conventional in their behavior but highly individualistic in spirit. Other studies by Helson, specifically those concerned with the relationships between mathematical creativity and personality, are discussed in a later section of this review.

Masculine Identification Hypothesis. The existence of sex differences in mathematical performance has been interpreted from a psychoanalytic viewpoint in the "masculine identification hypothesis" of Plank and Plank (1954). These writers claim that mathematics is primarily a masculine enterprise because a high level of controlled aggression is necessary for mathematical activity. Therefore, women who like mathematics tend to identify with a strong male figure. A sprinkling of both negative and positive results has been obtained in studies designed to test the masculine identification hypothesis. For example, Lambert (1960) found no correlation between mathematical proficiency and masculinity of interest in either sex. In actuality, the female mathematics majors in Lambert's study showed up as more "feminine" on the MMPI than non-mathematics majors. But Elton and Rose (1967) reported that college girls who were average in English but high in mathematics on the American College Tests had higher theoretical and lower esthetic interests (i.e., more "masculine" interests) than girls who were high in English but average in mathematics achievement. Consequently, these writers concluded that "masculine role" is an important factor to consider in predicting differences in ability in English and mathematics.

Mathematically Gifted Children

Special attention has been focused on the personalities of mathematically gifted children, who are an important human resource. As with gifted children in general, the mathematically gifted tend to be curious, persistent, highly intelligent, and equipped with good memories (Duncan, 1961). They are also reluctant to accept the obvious, and manifest a positive dislike for repetitiousness and routine (Srinivason, 1968).

It has been observed that as youngsters the mathematically gifted have a keen interest in mathematics and work at this subject with pleasure and with-

out compulsion (Krutetskii, 1969d). In elementary school they are typically well-adjusted, flexible, and realistic, but highly independent and frequently unconventional in their thoughts. However, when their independence of thought and action is threatened they tend to respond with hostility and self-assertion (Haggard, 1957). In high school the mathematically gifted tend to be concerned with theoretical rather than social issues, energetic and competitive, somewhat unconventional, but usually quite well-adjusted (Kennedy et al., 1960; Kennedy & Walsh, 1965).

The Personalities of Creative Mathematicians

Among the personality variables related to creativity in various fields are independence of judgment, unconventionalism, a preference for complexity in phenomena, a greater openness to experience, less defensiveness, greater freedom from restraints, and a high energy level (Barron, 1955; MacKinnon, 1962). McClelland (1963) also noted that creative individuals are more willing to take risks, which may help account for their success in discovering novel solutions. On the other hand, the creative individual is not impulsive. By being better able to tolerate ambiguity and engaging in continuing self criticism (Tumin, 1962; McGannon, 1972), he can reject the facile, conventional solution to a problem in favor of a more creative and useful goal.

Many observations and anecdotes bearing on the personalities of famous mathematicians have been recorded in biographies and other sources. Albert Einstein, for example, was a terribly shy, modest man who was amazed that people were interested in what he had to say. In contrast, his friend Guido Fubini was self-centered, easygoing, enthusiastic, courageous, and quite aware of his own abilities (Dodge, 1969). Another famous mathematician, Guiseppi Peano, was, like Einstein and Fubini, dedicated to his work, but also somewhat

disorganized in interpersonal situations (Kennedy, 1968). Biographies and anecdotes about other mathematicians point to as many differences as similarities among the personalities of creative men in this field.

From time to time such characteristics as a fondness for music and other arts, a great interest in chess, a tendency to have unhappy marriages, and Anglo-Catholic religious preferences have been attributed to mathematicians (Hadamard, 1954; Smith, 1964). The typologist Ernst Kretschmer (see Smith, 1964) concluded that the majority of famous mathematicians or mathematical physicists (e.g., Copernicus, Napier, Kepler, Leibnitz, Newton) have displayed many of the nervous and eccentric traits, in addition to the formal, abstractive habits of thought, that characterize the schizothymic (schizoid) temperament.

More carefully designed empirical studies of the personalities of creative mathematicians have not yielded very impressive results. Walker (1952), for example, hypothesized that creative mathematicians and chemists manifest all of the following characteristics: originality, flexibility, fluency, ability to perceive disequilibria in the environment, and the ability to tolerate ambiguity. Although all of the 18 individuals studied by Walker showed some of these characteristics, only one showed all of them. A series of investigations by Helson (Helson, 1967, 1968; Helson & Crutchfield, 1970 a, b), inspired by certain psychoanalytic, typological concepts, will be considered in more detail.

Helson's Studies of Neumann's Types. The first two studies (Helson, 1967, 1968) were designed to test the hypothesis that creative men differ from others in a "patriarchal" cognitive style and that creative women differ from others in a "matriarchal" cognitive style. These two styles refer to Erich Neumann's theory of two types of consciousness—patriarchal and matriarchal. "Patriarchal

consciousness is assertive, objective, analytical, purposive. The intellect functions as an organ for swift registration, development, and organization. In matriarchal consciousness what is to be understood must first 'enter' in the sense of fructification. The psyche is filled and permeated with an emotional content over which it broods until an organic growth 'comes forth.'" (Helson, 1967, p. 214) The findings gave some support to the hypothesis, but wide individual differences among the creative mathematicians were also observed. In general, the creative men mathematicians were characterized as more assertive, outgoing, and enriched in personality than the creative women mathematicians or comparison subjects; they also possessed higher social status, more self-assurance, and participated in professional activities to a greater extent. In contrast, the creative women mathematicians were described as more introverted, reflective, receptive, and inner-oriented. A further study (Helson & Crutchfield, 1970b) explored variations in personality within a group of 34 creative mathematicians. The results of a cluster analysis of scores on a Mathematicians Q-Sort were interpreted as evidence for five personality types within this group; an "adapted" type, a "conflicted" type, two "patriarchal creative" types, and one "matriarchal creative" type.

Obviously the findings of the correlational, descriptive studies reviewed above are at best suggestive. Not only do they fail to account for the wide variation in creative abilities among mathematicians, but somewhat different results would undoubtedly be obtained on cross-validation. And even if personality assessment devices were vastly improved, situational and contingency factors undoubtedly play as large a role as personality traits in stimulating and fostering creativity. For example, what child psychologist, equipped with all of the personality assessment devices that

20th-century science has been able to devise and transplanted to the middle of the 17th century, could have predicted that the boy Isaac Newton would be one of the most creative geniuses of all time? Newton's genius, which did not manifest itself during his childhood, was encouraged and developed in large measure by his teacher Isaac Barrow. Teachers are, of course, not the only "situational" variables that help develop creative minds. In the case of Norbert Wiener (1953), a stern and demanding father played a crucial role.

Parental and Societal Influences

The influence of the father in the lives of men and women mathematicians has been described by several writers (Wiener, 1953; Carlsmith, 1964; Iacobacci, 1970). According to Carlsmith (1964), boys learn the conceptual, analytic approach, which is necessary for success in mathematics, from a close, harmonious relationship with their fathers. This "analytic approach" consists of discriminating among stimuli, pursuing a solution directly, and disregarding extraneous material. In contrast, the thought processes of females are described as more global, with less discrimination among stimuli and a greater influence from extraneous material. In almost all of the handful of cases in which women became famous as mathematicians (Hypatia, Maria Agnesi, Sonja Kovalevsky, Emmy Noether), the father's influence, or that of another male relative, was significant (Iacobacci, 1970).

Home Environment. The father, of course, is not the only parent, and investigations of mathematical creativity have also cited the mother as important in the child's creative development (Helson & Crutchfield, 1970a). Both the father and mother help to determine the psychological climate of the home. The homes of mathematically creative people tend to be higher than average in socioeconomic status, more often Protestant or Jewish than Catholic (Helson & Crutchfield, 1970a), and characterized by high family pressure for achievement and parental expectations of success (Drevdahl, 1964). These pressures and

expectations are directed especially toward the oldest child, particularly if it is a boy. Unlike what might have been anticipated from the literature relating creativity to a democratic home environment, the home environments of the majority of the mathematically talented high school students studied by Kennedy and Willcutt (1963) were described as somewhat autocratic, with high demands for achievement and a fixed regimen.

Other Social Groups. Highly creative people are not usually noted for their extroversion, being more interested in ideas than in social activities (Cattell, 1963). A preference for working alone rather than in a group is usually manifested by creative people, because their goals and impulses often conflict with those of others (Brown, 1962). As Carlton (1959) noted in her study of 14 famous mathematicians, however, men such as these are not anti-social creatures. They are usually quite eager to communicate with people whose interests and abilities are similar to their own. Thus, Carlton's mathematicians took an active part in organizing scientific societies, editing journals, participating in international congresses, and in corresponding.

Education for Mathematical Creativity

Suggestions for Mathematics Teachers

Although the technique of brainstorming (Osborn, 1953) assumes that group interaction exerts a positive influence of creativity, research findings reveal that pupils working in groups are seldom superior in creativity to those working alone (Spraker, 1960). In fact, Banghart and Spraker (1963) reported a slight but consistent advantage in solving problems alone. Therefore, since creative people tend to be rather independent, it has been suggested that teachers of creative students not stress group participation. Rather, they should provide opportunities for such students to work alone on their own interests. As indicated previously, gifted pupils dislike dull, routine work, but they are stimulated by opportunities to discover things on their own.

Since this is particularly true of older students, more memory work can be given in the early years rather than in the upper grades. Throughout the grades, following the presentation of basic concepts by the teacher, the mathematically gifted should be constantly encouraged to explore and discover for themselves. (Erhart, 1960; Meconi, 1967)

As McGannon (1972) recognizes, no one knows precisely how to organize experiences that will foster creativity. One important step, however, involves redesigning mathematics textbooks to present not only paradigms but real problems of the sort that professionals have to solve and the various solution techniques that they employ (Erhart, 1960; McGannon, 1972). Even more important than the textbook as a key to creative thinking in the mathematics classroom is the teacher himself. A creative teacher produces creative students. Such a teacher--by arranging the order in which topics are taken up, posing problems, asking questions, encouraging discussion, and providing opportunities for observing and exploring in the mathematics laboratory—creates an atmosphere that stimulates creative behavior (Romey, 1970). Carlton's (1959) description of the approaches to teaching of 14 famous mathematicians contains some relevant ideas. As teachers, these men were enthusiastic and personally interested in their students, providing them with many opportunities for personal initiative and the development of individual responsibility. This was accomplished by means of seminars, mathematics laboratories, research collaboration, and other situations in which the teacher acted as a stimulator and guide rather than merely a tutor. Also of value to the mathematics teacher, not only in working with the gifted but other students as well, is a list of suggestions made by Henderson and Pingry (1953) for developing the ability to solve problems. These suggestions include: encouraging questions, verbalization, and

hypothesis testing; constructing models and using heuristics; stressing relationships; shifting attention to break up incorrect mental sets; being patient and friendly.

Creative Mathematics in the Elementary School

Similar suggestions for the creative teaching of elementary school mathematics have been offered. Reed (1957) states that the creative mathematics teacher uses experience-oriented problems and encourages students to formulate hypotheses and think of solutions for themselves. Thus, the teacher places a high value on the child's creative efforts and knows when and how to assist these efforts. If the teacher respects the child and believes in his ability to think, the latter may make mathematical discoveries for himself. To illustrate this point, Cochran (1970) describes two pieces of mathematics—the graphing of a vertical line and an algorithm for subtraction—discovered by a second and a third grader, respectively.

During the past few years there has been an increasing emphasis on discovery learning at the elementary school level. Sealey (1960) maintained, for example, that children learn mathematics by discovering it for themselves through doing and using real things. To assist in discovery learning and inductive thinking, games and manipulative play activities of various kinds have been devised (see Dienes, 1963). Under the guiding hand of the teacher, and by means of group discussion, children are given the opportunity to make discoveries rather than being taught directly. Some of the impetus for the fostering of cognitive growth through the manipulation of concrete things and discovery has come from Piaget's theories of developmental stages (see Adler, 1966), as adapted and extended by Bruner and his associates (Bruner, (1966).

Illustrative of programs and investigations in elementary mathematics

learning that have utilized a discovery approach are the Madison Project (Davis, 1963, 1965) and various studies (Lanier, 1967; Richards & Bolton, 1971). Davis's (1963, 1965) educational orientation is eclectic; it emphasizes discovery, competitive games, and a variety of materials to build concrete images of important concepts, while discouraging the use of external reinforcement schedules and written tests. Although these procedures are viewed as having been generally successful in developing children's mathematical creativity as well as a liking for the subject, not all children benefit to the same degree. Not only does the effect of a discovery approach on performance vary with the pupil, but it also has different effects on different criterion variables. For example, in a study of three groups of British school children taught by a discovery approach, a traditional approach, or a balanced combination of the two, Richards and Bolton (1971) found that average scores on mathematics achievement tests were lower but scores on creative thinking were higher in the first group than in the other two groups.

Special instructional techniques designed to enhance either general creativity or mathematical creativity sometimes appear to have little if any effect on performance. For example, neither Broome (1968) nor Borgen (1971) observed significant differences in general creativity in pupils given special "creative" treatment. The fifth graders in Broome's study who were taught by "high creative" teachers were not statistically different from those taught by "low creative" teachers in gains on Torrance's Test of Creative Thinking. And the fourth through sixth graders in Borgen's study who attended schools designed to create independent, courageous people scored no higher on Guilford's creativity tests than students who attended regular schools during the year. Finally, Meyer (1970) attempted to encourage

individual mathematical creativity in first graders by the use of fifteen 20-minute lessons, each consisting of open-ended geometry problems (triangles) with a moderate amount of structure. Again, however, significant differences in general creativity or mathematical creativity were not found.

Various explanations for the negative results obtained in these studies and the low generalizability of the findings are possible: high initial levels of criterion performance, small sample sizes, failure to randomly assign subjects to subgroups, lack of proper control over extraneous variables, inadequate measuring instruments. With respect to the last item, the reliabilities and validities of psychological tests and inventories are typically lower in the early grades than further up the educational ladder. But, unfortunately, the shortcomings of research on creativity are not limited to the elementary school level.

Creativity and High School Mathematics

Moore and Cain (1968) found significant changes in the scores on selected logical reasoning and creativity tests in Guilford's battery with two high school algebra classes exposed to a new mathematics curriculum. This investigation suffered, however, from the lack of a separate control group. On the other hand, both Clark (1967) and McGannon (1971) employed control groups but the subjects were not randomly selected or randomly assigned to the two groups. Clark's experimental group was given no explanation, but were expected to explore the mathematics problems presented on their own. During class time the students discussed the problems and evaluated different solutions to them. Meanwhile, the control group was taught in a conventional, teacher-centered manner. Although the controls solved more story problems than the experimentals during the course, the latter group showed greater gains in problem-solving ability. Furthermore, post-test

achievement scores in the experimental group were correlated with both creative and academic ability, but only with academic ability in the control group. McGannon's (1971) subjects were high school students enrolled in advanced placement courses in calculus. One class used a rigorous calculus textbook, and the other an intuitive calculus textbook. An aptitude-treatment interaction was obtained, in that students who were classified as creative on Mednick's Remote Associates Test and taught by the intuitive method made significantly higher calculus achievement scores than the non-creatives taught by either the rigorous or the intuitive method or the creatives taught by the rigorous method.

Kinsella (1970), addressing himself to the teaching of problem solving in the secondary school, has made a number of suggestions that should be helpful to high school mathematics teachers. All of these suggestions stem from the premise that one learns to solve problems by solving them. Referring to Polya's (1965) use of heuristics, Kinsella (1970) states that the student should be encouraged to keep these questions in mind: Have you seen a problem like this before? Can you solve a special case? Would it help if you dropped part of the condition for a while and worked at the rest? Have you used all of the data? Guess an answer. Do you know a more general problem? Furthermore, it is suggested that the teacher break up "sets" by having students ask questions, rewarding unique explanations and different ways of solving the same problem, and urging students to generalize results.

In addition to enrichment, special instruction, and differentiated courses for mathematically creative students in high school, acceleration is sometimes recommended in the case of the highly gifted (Duncan, 1961). Many universities permit academically talented high school students to enroll in courses or enter college before they have graduated from high school. Even when such students

are admitted to college from junior high school they typically do well in college mathematics and science courses when compared with other college students (see Keating & Stanley, 1972).

Creativity and College Mathematics

At the college level, as in elementary and high school, the personality and techniques employed by mathematics teachers have a crucial effect on the attitudes and achievements of their students. One of the findings of a National Science Foundation survey of creative mathematicians (Albert, 1957) was that the field of concentration of these men was greatly influenced by teachers, and especially college teachers.

Recognizing that prospective mathematics teachers will shortly be in charge of the mathematics education of a large number of students, new approaches to teaching the subject are frequently tried out first on these prospective teachers. The competencies, attitudes, and interests of teachers certainly affect the motivations and abilities of students, so efforts must be made to train highly motivated and capable mathematics teachers. This is a goal of Gibb's (1970) modules for teaching creative problem solving to potential mathematics teachers. Through the use of these modules ("Application of Deferred Judgment," "Recognizing the Real Problem", and "Developing the Solution"), prospective teachers are given practice in brainstorming, problem redefinition, generating and evaluating solutions, and overcoming mental sets.

Gibb (1970) has reportedly had some success in teaching creative problem solving by this modular approach. McCormack (1970) has used similar techniques (brainstorming, morphological analysis of problems, inquiry development sessions, written creative thinking exercises) in training students in science methods courses to be creative. More specific to mathematics education,

Baur (1971) and Buckeye (1968) have compared conventional (lecture) classes in mathematics for teachers with more open-ended, creativity-oriented classes. Buckeye describes the atmosphere of his creative classes as encouraging and respecting students' questions and imaginative or unusual ideas, while demonstrating to them that their ideas have value. In both Baur's and Buckeye's experiments the creative classes showed a greater mean change in mathematical creativity than the conventional classes.

In another experiment, Buckeye (1970) studied the effects of a laboratory approach in mathematics methods classes. This approach emphasized discovery through concrete experiences and activities of various kinds. The finding was that the laboratory approach increased general creativity, improved attitudes toward mathematics, and had no detrimental effect on mathematics achievement. Somewhat different results were obtained by Mainville (1972) in an experiment that compared two groups of prospective elementary teachers in terms of their scores on a test of mathematical creativity. Group 1 received a lecture-textbook presentation in a pre-service mathematics course, whereas with Group 2 mathematics activity materials were also used. Although both groups increased from pre- to post-test in mean score on the mathematical creativity test, the mean gains of the two groups were not significantly different.

Recommendations for Further Research

The methodological criticisms that can be made of the investigations that have been reviewed above are obviously not unique to research on mathematical ability and creativity. Many educational studies suffer from small and biased samples, inadequate controls, failure to meet the assumptions underlying specific statistical methods, and unreliable or invalid

measuring instruments. In addition, the meaning of the concept of creativity is unclear and varies somewhat with the investigation and the technique. Finally, high initial levels on the criterion variable may vitiate any effect that an experimental variable has on the criterion. Since the authors of studies that have been reviewed were undoubtedly aware of these methodological pitfalls of research in education, it would be tedious to dwell on them. Rather, we shall consider some of the most promising directions for further research on mathematical ability and creativity.

Measurement

Clearly, the value of research findings is severely limited by inadequate measuring instruments. With regard to creativity in general and mathematical creativity in particular, there is an obvious need for better tests and inventories. Carlton (1959) and others have provided inventories of various creative characterisitics or behaviors that might serve as a starting point for instrument development. Also, the reader will recall that certain studies employed measures of "general creativity," whereas others used specially designed measures of "mathematical creativity," frequently with no clear explanation of why one instrument rather than another was selected. This indicates a need for investigations aimed at differentiating between creativity in general and creativity in mathematics. A factor analysis of various measures of mathematical and general creativity, perhaps in combination with other ability tests, at various age levels and for both sexes, could help to clarify what is being measured by some of the available instruments.

Developmental Studies

Some of the most exciting and potentially contributory research in

education and psychology is being conducted in the area of human development. Observations and tape recordings of children's verbalizations during problem solving are yielding insights into the thinking and creative processes (Rimoldi, Aghi, & Burder, 1968; Krutetskii, 1969 b, c). It will be recalled that, from his observations of mathematically gifted children, Krutetskii concluded that some people's nervous systems are more sensitive to relations, symbols, and numbers, and that associations involving them are formed more easily and with less effort and greater retention. These hypotheses may serve as a stimulus for systematic research on psychophysiological correlates of mathematical ability. Individual differences in physiological and behavioral responses during mathematical problem solving and in the perception, learning, and retention of mathematical stimuli should be carefully analyzed as a test of Krutetskii's hypotheses.

Another potentially fruitful line of research is concerned with the interaction between heredity and environment in the production and differentiation of mathematical abilities. For example, studies of the interaction between age and sex on mathematical abilities (Dye & Very, 1968) should be extended to younger children, different socioeconomic and ethnic groups, and to longitudinal investigations of changing factor structure in a given group. In particular, the effects of education on the factorial structure of mathematical abilities needs to be looked at more closely as a test of Ferguson's (1956) positive transfer hypothesis. Furthermore, Stafford's (1972) recent review of hereditary components in quantitative reasoning points to a need for further research of the sex-linked, recessive-gene model and the impact of specific environmental experiences on individual placement along the two hypothetical ability distributions referred to in that review.

In addition to these developmental studies of mathematical ability, the effects of age, sex, social class, ethnic group membership, and other organismic variables on creativity in mathematics require consideration. The design of instruments or observational techniques for the early identification of mathematical creativity in children is another potentially useful task.

Creativity and Teaching

Accepting the premise that the teacher is the key to creative thinking in the classroom, it is surprising that we know so little about what specific teacher behaviors or activities really make a difference in promoting creative responses in pupils. In spite of this fact, Gibb (1970) and others have designed approaches that aim to increase creative behavior in prospective teachers and, as a consequence, creative behavior in their students. In this context, a popular research design is to compare an "intuitive," "discovery," or "creative" instructional treatment with a "traditional," "conventional," or "teacher-centered" treatment. Although perhaps commendable, such special methods of teaching creative thinking and problem solving require careful evaluation. Not only is there a need to determine whether these methods affect the prospective teacher's short-term behavior, but, more significantly, whether these effects carry over into the classroom situation. Meanwhile, analyses of specific teacher behaviors that promote creative responses in pupils must be continued.

Even the most inspiring, creative teacher, employing the most generally effective instructional techniques, will not have an equal influence on all students. As documented by Behr (1967), McGannon (1971) and others, method of instruction interacts with learners' abilities in its effects on perfor-

mance. Consequently, another promising direction for research is concerned with the interactions of mathematical creativity and ability with teacher characteristics and instructional method in their influences on scores on various criteria of mathematical performance.

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